Global Modeling
with NASA Supercomputing Technology:
When Sandy Meets Lorenz

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Outline

1. Introduction
3. Supercomputing and Visualization Technology
4. Simulations of Hurricanes Sandy and Others
5. Nonlinear Processes in Modified Lorenz Models
6. Summary and Future Tasks
Lorenz: Chaos Theory with Butterfly Effect

- The butterfly effect of first kind: *sensitive dependence on initial conditions*.
- The butterfly effect of second kind: *a metaphor for indicating that small perturbations can alter large-scale structure*.
- Lorenz’s studies suggested finite predictability and nonlinearity as the source of chaos.

The studies by Lorenz (1963, 1972) laid the foundation for chaos theory, which was viewed as the third scientific revolution of the 20th century after relativity and quantum mechanics (e.g. Gleick, 1987; Anthes 2011).
Sandy: Tropical Cyclones with Multiscale Interactions

Hurricane France
Hurricane Howard
Tropical Storm Phoebe
Super typhoon Sonda
When Sandy Meets Lorenz

<table>
<thead>
<tr>
<th>Sandy (2012)</th>
<th>Lorenz (1963)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hurricane Dynamics</td>
<td>Chaos Dynamics</td>
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<tr>
<td>Earth Science</td>
<td>Mathematics</td>
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<tr>
<td>Complex Model</td>
<td>Theoretical Model</td>
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<td>Idealized Model</td>
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<td><strong>PDE</strong></td>
<td><strong>ODE</strong></td>
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<td>Complexities in Modeling</td>
<td>Complexities in Mathematics</td>
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<td>Multiscale Processes</td>
<td>Butterfly Effect</td>
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<tr>
<td>PEEMD</td>
<td>SAT</td>
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High-impact Tropical Weather: Hurricanes

Each year tropical cyclones (TCs) cause tremendous economic losses and many fatalities throughout the world. Examples include Hurricanes Katrina (2005) and Sandy (2012).

Hurricane Katrina (2005) (Shen et al., 2006b; 2013a)
- Cat 5, 902 hPa, with two stages of rapid intensification
- The sixth-strongest Atlantic hurricane ever recorded.
- The third-strongest landfalling U.S. hurricane ever recorded.
- The costliest Atlantic hurricane in history! ($100+ billion)

Hurricane Sandy (2012) (Shen et al., 2013c)
- The deadliest and the most destructive TC of 2012 Atlantic hurricane season
- The second-costliest hurricane in United States history ($50 or 75$ billion)
- The largest Atlantic hurricane on record
Early Efforts with Global Models
(1999~2003)


- **Unix System and Network Programming**: Unix curses, device control (serial I/O), file system control, inter-process communication (pipes, semaphore, shared memory, TCP/IP sockets), process control, signal handling.
- **System Administration**: Unix/Linux/MS windows system installation.
- **Supercomputing (Parallel/Distributed/Cluster Computing)**: MPI (Message Passing Interface), MPI-2 remote memory access, MLP (Multi-Level Parallelism), OpenMP, ESMF (Earth Science Modeling Framework), POSIX Threads, and JAVA Threads. Knowledge of Grid computing.
- **Software**: Fortran (F77/F90/F95), OOP (Object Oriented Programming), C/C++, JAVA, Basic, Pascal, UNIX Shells, UNIX m4 script, PERL, Python, PHP, HTML, XML, XHTML, CGI, AWK. CVS (Concurrent Version System), GNU Make, gdb, LaTex, MATLAB, VMWARE, Secure Shell, MS-Office, VIS5D, AVS, GrADS, NCAR Graphics, GEMPAK.
- **Numerical Models**: MM4, MM5, ARPS, WRF, NASA GEOS-4, GEOS-5 (beta), NCAR CAM, MMF
My Journey with Computational Science

08. 02. 2004 – Initial Columbia Promising

NASA's new Columbia system has already produced results that foretell breakthrough scientific achievements even before the system is completed. Bo-Wen Shen at Goddard Space Flight Center has obtained accurate hurricane tracking and prediction, at increased (0.25 degree) resolution, with up to a 72-hour forecast.
Computational Science

- **Computational Science** (CS) is defined as an inter-disciplinary field with the goals of understanding and solving complex problems using high-end computing facilities.
- CS is identified as one of the most important fields of the 21st century to contribute to the scientific, economic, social and national security goals of USA by the President’s Information Technology Advisory Committee (PITAC).
NASA Supercomputing and Visualization Systems

Pleiades Supercomputer (as Nov. 2014)
- one of a few petascale supercomputers
- $R_{\text{max}}$ of 3,375 teraflops (LINPACK);
  $R_{\text{peak}}$ of 3,988 teraflops
- 160,768 cores in total; Intel Xeon processors, Nehalem, Westmere, Sandy Bridge, Ivy Bridge
- 532 TB memory
- 3.1 PB disk space
- Largest InfiniBand network.

- Large-scale visualization system
  - 8x16 LCD tiled panel display
  - 245 million pixels
- 128 nodes
  - 1024 cores, 128 GPUs
- InfiniBand (IB) interconnect to Pleiades
  - 2D torus topology
  - High-bandwidth

When Sandy Meets Lorenz

National Central Univ. Jan. 16, 2015
My Journey with Computational Science

2. Research results featured by Dr. Jack Kaye (Associate Director at NASA/HQs) at the Interdepartmental Hurricane Conferences in 2012, 2013, and 2014.
3. Research results featured in a recent President's Corner article of UCAR Magazine by Dr. Rick Anthes in 2011. The article is entitled “Turning the Tables on Chaos: Is the atmosphere more predictable than we assume?”
4. Research results featured in NASA News Stories (07/2010 and 11/2010). It was also translated in Chinese by Science and Technology Division, Taipei Economic and Cultural Representative Office in the United States.
5. Research results appeared in news medias, such as MSNBC, PhysOrg.com, National Geographic--Indonesia, ScienceDaily, EurekAlert, Yahoo News, TechNews Daily, Scientific Computing, HPCwire, etc. (2010)
7. Selected as Journal Highlights by American Geophysical Union (07/2006)

“Is new science being produced or just really cool pictures?” which was raised by Mahlman and others who have reservations
Hi Bowen,
I have gone through some of your presentations and note with special interest your comments on scale interactions and predictability of tropical cyclones. I did some work closely related to this in the 1980s, and hypothesized that some mesoscale systems of importance were predictable far in advance if the proper large-scale conditions were known. I put these papers in a folder on my webshare at www.fin.ucar.edu/antheswebshare/ and a summary of them is attached. You might find some of these ideas from 25 years ago interesting. I think your recent work is confirming my hypotheses and thoughts, and I am glad to see this! The key to accurate prediction of tropical cyclogenesis is the get the right large-scale fields, have sufficient resolution for TC spinup, and appropriate physics!
Rick

--
Dr. Richard A. Anthes
1. Model Dynamics and Physics:

- The finite-volume dynamical core (Lin 2004)
- The NCAR physical parameterizations, and NCEP SAS as an alternative cumulus parameterization scheme
- The NCAR land surface model (CLM2, Dai et al. 2003)

2. Computational design, scalability and performance (suitable for running on clusters or multi-core systems)


Physics Parameterizations

- Moist physics:
  - Deep convections: Zhang and McFarlane (1995);
    Pan and Wu (1995, aka NCEP/SAS)
  - Shallow convection: Hack (1994)
  - Large-scale condensation (Sundqvist 1988)
  - Rain evaporation

- Boundary Layer
  - First order closure scheme
  - Local and non-local transport (Holtslag and Boville 1992)

- Surface Exchange
  - Bryan et al. (1996)

General Issues in Global Modeling

• Hydrostatic vs. non-hydrostatic (resolved scale ~ 10km, Pielke 2002, Shen 1992)

• Cumulus Parameterizations (validity at a mesoscale resolution? Validity for TC formation?)

• (Additional) Required Physical Processes (e.g., surface flux exchange for TC formation and intensification)?

• Added skill in weather simulations?
### Grid Cells vs. Grid Spacing

<table>
<thead>
<tr>
<th>Resolution</th>
<th>x</th>
<th>y</th>
<th>Grid cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1° (~110km)</td>
<td>288</td>
<td>181</td>
<td>52 K</td>
</tr>
<tr>
<td>0.5° (~55km)</td>
<td>576</td>
<td>361</td>
<td>208 K</td>
</tr>
<tr>
<td>0.25° (~28km)</td>
<td>1000</td>
<td>721</td>
<td>721 K</td>
</tr>
<tr>
<td>0.125° (~14km)</td>
<td>2880</td>
<td>1441</td>
<td>4.15 M</td>
</tr>
<tr>
<td>0.08° (~9km)</td>
<td>4500</td>
<td>2251</td>
<td>10.13 M</td>
</tr>
<tr>
<td>MMF (2D CRM)</td>
<td>144x64</td>
<td>90</td>
<td>829 K</td>
</tr>
</tbody>
</table>

The 1/12 degree model with 48 vertical levels has 480 M grid points. In comparison, the hyperwall-2 is able to display 245 M pixels.

1/8 degree, 02:39:51/5day using 1440 cores.

1/4 degree, 35 minutes/ 10 days run using 240 cores.
Concurrent Visualization: Why and How?

1. Large time-varying simulations generate more data than can be saved
   - Problem gets worse as processing power increases
   - Models increase spatial and temporal-resolution
2. Saving data to mass storage consumes a significant portion of runtime
3. Only a small fraction of timesteps are typically saved and important dynamics may be missed

   process huge data efficiently

1. Extract data directly from running simulation for asynchronous processing
   • Add instrumentation to the simulation code, usually quite minimal
2. Simultaneously produce a series of visualizations
   • Many fields;       • Multiple views
3. Generate and store images, movies, and “extracts”
4. Send visualizations of current simulation state almost anywhere, including web
   • Images of current state kept up-to-date in web browser
   • Stream progressively growing movies to remote systems
5. Use hyperwall-2 for parallel rendering and asynchronous I/O

   generate visualizations while model is still running
Concurrent Visualizations: GMM


Concurrent Visualizations: WRF
Training and Stabilizer Wheels

In the (regional) models, the interaction is one way.

- Heavy Duty BMX Training Wheels for 20-Inch BMX Wheel Bicycles, boys or girls frame
- Heavy-Duty Steel Tubing to Support over 200lbs! Do not settle for unsafe universal axle mounted training wheels
Coupling: Training Wheel and Bicycle Trailer

Its own memory?
Architecture of the CAMVis v1.0
(the Coupled Advanced Multiscale modeling and concurrent Visualization systems; Shen et al. 2011)
During the past twenty years, track forecasts have been steadily improving (left panel), but intensity forecasts have lagged behind until recently (e.g., 2012) (right panel).

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... the general problem of tropical cyclogenesis remains in large measure, one of the greatest mysteries of the tropical atmosphere.”
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Although some aspects of the transformation of atmospheric disturbances into tropical cyclones are relatively well understood, the general problem of tropical cyclogenesis remains in large measure, one of the greatest mysteries of the tropical atmosphere.” – Kerry Emanuel, The Divine Wind (2005)
Formation of Hurricane Helene (2006)

- Upper-level winds in red; middle-level winds in green; low-level winds in blue
- Low-level CC (cyclonic circulation); Upper-level AC (anticyclonic circulation)
Simulations of Helene (2006) between Day 22-30
(Helene: 12-24 September, 2006)

Track Forecast

Intensity Forecast

OBS

model

OBS

to what extent can large-scale flows (e.g., an AEW) determine the movement and intensification of Hurricane Helene?

Remarkable simulations of TC formation and different tropical waves include:

- TC Nargis (2008) and an Equatorial Rossby (ER) Wave (Shen et al., 2010a)
- Hurricane Helene (2006) and an African Easterly Wave (AEW; Shen et al., 2010b)
- Twin TCs (2002) and a mixed Rossby Gravity (MRG) Wave (Shen et al., 2012)

The Wall Street Journal
October 28, 2012

AGU Geophysical Research Letter
September 19, 2013

**Genesis of Hurricane Sandy (2012) simulated with a global mesoscale model**

B.-W. Shen,¹,² M. DeMaria,³ J.-L. F. Li,⁴ and S. Cheung⁵

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Article first published online: 19 SEP 2013 | DOI: 10.1002/grl.50934

**Key Points**

- A GMM produced a remarkable 7-day track and intensity forecast of TC Sandy
- Sandy's genesis was realistically simulated with a lead time of up to six days
- The lead time is attributed to the improved simulations of multiscale systems
7-day Simulation and Visualization of Sandy

Collaboration with Dr. David Ellsworth of NASA/ARC/NAS

Figure 7: 4D visualizations of Hurricane Sandy (2012) at 00Z Oct. 23 (a), 12Z Oct. 25 (b), 12Z Oct. 27 (c), and 12Z Oct. 28 (d). During the period, Sandy (labeled in a pink ‘S’) moved northward under the influence of the sub-tropical middle- and upper-level trough (to Sandy’s northwest) (a), interacted with the trough that was deepening (b), increased its spatial extent (c), and encountered a pair of high-and-low blocking pattern over the North Atlantic, which prevent Sandy moving eastward further (d).
Questions

• From a modeling perspective, why high-resolution global models have skills?

• From a perspective of chaos (nonlinear) dynamics, are the simulations of TC formation consistent with chaos theory? (e.g., sensitive dependence on initial conditions)? → a high-order Lorenz model (Shen 2014a,b)

• From a perspective of hurricane dynamics, if and how the lead time of hurricane predictions can be extended? → a conceptual model and approaches
1. to what extent can large-scale flows determine the timing and location of Tropical Cyclone (TC) genesis? (e.g., downscaling)

2. to what extent can resolved small-scale processes impact solutions’ stability (or predictability)? (e.g., upscaling)
Scientific Goals

Large scales

1. to what extent can large-scale flows determine the timing and location of Tropical Cyclone (TC) genesis?
   (e.g., downscaling)

2. to what extent can resolved small-scale processes impact solutions’ stability (or predictability)?
   (e.g., upscaling)

Medium/Meso scales

Small scales

MAP: Multiscale Analysis Package
PEEMD: Parallel Ensemble Empirical Mode Decomposition
SAT: Stability Analysis Tool
To improve the prediction of TC's formation, movement and intensification, we need to improve the understanding of nonlinear interactions across a wide range of scales, from the large-scale environment (deterministic), to mesoscale flows, down to convective-scale motions (stochastic).
Data and Model Analysis with the MAP

Tropical Waves → MAP with PEEMD → Tropical Cyclone Formation → MAP with SAT → small-scale processes

Multiscale (Data) Analysis  Model Analysis
Empirical Mode Decomposition (EMD)

1. HHT (Hilbert Huang Transform, Huang et al., 1998) consists of Empirical mode decomposition (EMD) and Hilbert Transform.

2. The data-driven EMD method is Complete, Orthogonal, Local, and Adaptive (COLA), which is ideal for the local and nonlinear analysis.

3. EMD generates a set of intrinsic mode functions (IMFs), each of which has features with comparable scales (Wu and Huang 2009, and references therein).

4. EMD performs like a filter bank (e.g., a dyadic filter); the unique feature suggests a potential for hierarchical multiscale analysis.


PEEMD: Scaling of 5000 Cores

MRG Case, Grid:1000x1000 (400MB), Ivy Bridge Processors

4-Level Parallelization
SGI MPT library is used

<table>
<thead>
<tr>
<th>Grid DELayout</th>
<th>I</th>
<th>J</th>
<th>Ens</th>
<th>OMP</th>
<th>Total</th>
<th>Time (secs.)</th>
<th>Speed Up</th>
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<td>6</td>
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<td></td>
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<td>50</td>
<td>50</td>
<td>2</td>
<td></td>
<td>5000</td>
<td>123.85</td>
<td>52.8</td>
</tr>
</tbody>
</table>

Parallel efficiency:
- 2000 cores, 28.2/(2000/60)=84.6%
- 5000 cores, 52.8/(5000/60)=63.4%
Analyzing Real-world Cases with the PEEMD

Multiscale analysis of the following case is discussed:

- Hurricane Sandy (2014) and tropical waves (e.g., MRG and ER)

1. to what extent can large-scale flows determine the timing and location of TC genesis? (e.g., downscaling)
Decompositions of $V$ winds with the PEEMD
Characteristics of Wave-like Disturbances (200-hPa V winds)

- A dispersion relation describes the relationship between the wavelength and period of a specific wave.
- A mixed Rossby gravity (MRG) wave is indicated by dashed lines.
- An Equatorial Rossby (ER) wave is indicated by white lines.
- X indicates the location of Sandy.

(Shen et al., 2013c, GRL; Shen et al., 2013e, IEEE Earthzine).

\[
\sigma = \frac{k \sqrt{g H_e}}{2} \left[ 1 - \left( 1 + \frac{4\beta}{k^2 \sqrt{g H_e}} \right)^{1/2} \right],
\]

\[
\sigma = \frac{-\beta k}{k^2 + \beta (2n + 1) / \sqrt{g H_e}}.
\]
Data and Model Analysis with the MAP

Tropical Waves  \[\xrightarrow{\text{MAP with PEEMD}}\] Tropical Cyclone Formation  \[\xrightarrow{\text{MAP with SAT}}\] small-scale processes

Data Analysis  Model Analysis
Scientific Goals

2. to what extent can resolved small-scale processes impact solutions’ stability (or predictability)?
   (e.g., upscaling)

• Increase or decrease complexities of the Lorenz model (3DLM) by deriving high-order Lorenz models (5DLM and 6DLM) or non-dissipative Lorenz model (NLM)
• Apply the SAT to examine the stability of the above modified Lorenz models with the aim of understanding the impact of increased degree of nonlinearity, dissipation or heating terms on solutions’ stability (Understanding the role of nonlinearity in chaotic responses)
• Investigate the possibility of applying the SAT (e.g., the calculations of eLE) to determining the predictability of global models
Nonlinearity and Forcing Terms
Rayleigh-Bénard Convection

By assuming 2D (x,z), incompressible and Boussinesq flow, the following equations were used in Lorenz (1963)

\[
\frac{\partial}{\partial t} \nabla^2 \psi = -J(\psi, \nabla^2 \psi) + \nu \nabla^4 \psi + g \alpha \frac{\partial \theta}{\partial x},
\]

\[
\frac{\partial}{\partial t} \theta = -J(\psi, \theta) + \frac{\Delta T}{H} \frac{\partial \psi}{\partial x} + \kappa \nabla^2 \theta,
\]

Here \(\psi\) is the streamfunction that gives \(u = -\psi_z\) and \(w = \psi_x\). \(\theta\) is the temperature perturbation. The constants, \(g\), \(\alpha\), \(\nu\), and \(\kappa\) denote the acceleration of gravity, the coefficient of thermal expansion, the kinematic viscosity, and the thermal conductivity, respectively.

- Navier-Stokes equation with constant viscosity
- Heat transfer equation with constant thermal conductivity
Lorenz Models

D, H, and N refer to as the dissipative terms, the heating term, and nonlinear terms associated with the primary modes (low wavenumber modes), respectively. \( D_s, H_s, \) and \( N_s \) refer to as the dissipative terms, the heating term, and nonlinear terms associated with the secondary modes (high wavenumber modes), respectively. NLM refers to the non-dissipative Lorenz mode.

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>H</th>
<th>N</th>
<th>D_s</th>
<th>H_s</th>
<th>N_s</th>
<th>Critical points for (X,Y)</th>
<th>( r_c )</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Linearzied 3DLM</td>
<td>V</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( X_c = Y_c = \pm \sqrt{b(r-1)} )</td>
<td>24.74</td>
<td>“1” Unstable as ( r &gt; 1 )</td>
</tr>
<tr>
<td>3DLM</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
<td>( (X_c, Y_c) = (\pm \sqrt{2\alpha r}, 0) )</td>
<td>42.9</td>
<td>conservative</td>
</tr>
<tr>
<td>3D-NLM</td>
<td>V</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5DLM</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>( X_c = Y_c \sim \pm \sqrt{2b(r-1)} )</td>
<td>41.1</td>
<td></td>
</tr>
<tr>
<td>6DLM</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
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</table>


Shen, B.-W., 2014c: Nonlinear Feedback in a Six-dimensional Lorenz Model. Impact of an Additional Heating Term. (to be submitted to JAS)
3DLM vs. 5DLM

- The butterfly effect of first kind: *sensitive dependence on initial conditions.*
- The butterfly effect of second kind: *a metaphor (or symbol) for indicating that small perturbations can alter large-scale structure.*
- Lorenz’s studies suggested finite predictability and nonlinearity as the source of chaos.
- Increased degree of nonlinearity (e.g., multiscale interactions) can stabilize solutions and thus improve simulations (Shen et al., 2014a,b).

The studies by Lorenz (1963, 1972) laid the foundation for chaos theory, which was viewed as the third scientific revolution of the 20th century after relativity and quantum mechanics (e.g. Gleick, 1987; Anthes 2011).
Major Results with the 5DLM

- **Generalized LMs:** We derived the generalized 5D and 6D LMs to investigate the impact of three higher-wavenumber modes on the numerical predictability.

- **Unique Features of the 5DLM:** The 5DLM is the lowest order generalized LM with improved system stability. The analytical solutions of the critical points in the 5DLM and its mathematical simplicity make it easier to identify the major feedback process and its role in the solutions' stability of the generalized LMs; and perform stability analysis near the critical points over a wide range of values in parameters ($\sigma$, $r$).

- **Nonlinear Feedback Loop:** Just as Lorenz demonstrated the association of the nonlinearity with the existence of the non-trivial critical points and strange attractors in the 3DLM, we emphasized the importance of the nonlinearity in both producing new modes and enabling subsequent negative feedback to improve solution stability.

  \[
  \frac{dY}{dT} = -XZ + rX - Y \quad \frac{dZ}{dT} = XYZ - XY_1 - bZ
  \]

- **Degree of Nonlinearity:** Inclusion of the M5 and M6 modes can extend the existing feedback loop and enable the subsequent negative feedback that stabilize the solution.
A Brief Summary of High-resolution Lorenz Models

1. The 3DLM contains nonlinearity, heating, and dissipative terms. (by introducing some of above terms, additional modes can change the stability of existing critical points and/or introduce additional critical points)

2. Two simplified 3DLMs include (i) nonlinearity only or (ii) nonlinearity and a heating term (appearance of a saddle points). → sources of chaos;

3. The 5DLM increases the degree of nonlinearity (with additive dissipative terms). → negative nonlinear feedback → improved stability;

4. The 3DLMP with a parameterized dissipative term produces solution’s stability comparable to that in 5DLM (a comparable equilibrium state) but different time evolution of solutions (a different transient solution); coarse resolution runs may produce a comparable climate (but different weather).

5. The 6DLM introduces an additional heating term, →(slightly) positive nonlinear feedback; excessive precipitation in high resolution runs may indicate appearance of chaotic responses → additional “smoothing terms” may be added to stabilize solutions by some modelers.

Additional modes in the 5DLM do not introduce additional critical points; a comparison of the 6DLM with the 5DLM does not suggest significant changes in the characteristics of critical points. Shen (2014a, b, c).
Track Forecasts of Sandy (2012) by different models

- Observation in white
- ECMWF in coral
- GFS in cyan
- GFS ensemble in yellow
- TVCA model consensus in red

Model forecast tracks for Sandy at 0000 UTC 23 October (a), 0000 UTC 24 October (b), 0000 UTC 25 October (c), and 0000 UTC 26 October (d). Solid color lines are the forecasts through 72 h, while dashed lines are from 72-120 h, and dotted lines represent the 120-168 h forecasts (top panels only). The ECMWF is in coral, the GFS ensemble in yellow, the GFS is in cyan, and the TVCA model consensus is in red.

Questions

• What are the controlling factors in determining the eastward or westward movement of Hurricane Sandy in the models?

• What are the characteristics of solutions near a saddle point? What are the factors in determining the accuracy of solutions near the saddle point?

• Is nonlinearity a source of chaos? (if so, is it good to increase a model’s resolution?) (additional fixed points?)

Nonlinearity can lead to diverged trajectories at a finite time in a non-dissipative Loren model with r=0 or r≠0.
The 3D non-dissipative Lorenz Model (3D-NLM)

To examine the role of the nonlinearity in the original 3DLM, our approach is to go back to simplify the original governing equations (for 2D Rayleigh-Benard convection) by dropping the dissipative terms, leading to

\[
\frac{\partial}{\partial t} \nabla^2 \psi = -J(\psi, \nabla^2 \psi) + g \alpha \frac{\partial \theta}{\partial x} + \nu \nabla^4 \psi,
\]

\[
\frac{\partial \theta}{\partial t} = -J(\psi, \theta) + \frac{\Delta T}{H} \frac{\partial \psi}{\partial x} + \kappa \nabla^2 \theta,
\]

which contain two kinds of “forcing” terms, nonlinearity and heating.

www.nonlin-processes-geophys-discuss.net/1/519/2014/
doi:10.5194/npgd-1-519-2014 (also being reviewed by the journal NPG; Sep. 17, 2014)
3D-NLM vs. 3DLM

**3D-NLM**

\[
\begin{align*}
\frac{dX}{d\tau} &= \sigma Y, \\
\frac{dY}{d\tau} &= -XZ + rX, \\
\frac{dZ}{d\tau} &= XY.
\end{align*}
\]

**3DLM**

\[
\begin{align*}
\frac{dX}{d\tau} &= -\sigma X + \sigma Y, \\
\frac{dY}{d\tau} &= -XZ + rX - Y, \\
\frac{dZ}{d\tau} &= XY - bZ.
\end{align*}
\]

*3D-NLM: 3D non-dissipative Lorenz model*

*3DLM: 3D Lorenz model*
Energy Conservation Laws

\[
\frac{d^2X}{d\tau^2} + M^2X = 0, \quad \bar{KE} + \bar{PE} = C_o \left( \frac{X^2}{2} - \sigma Z \right) = C_1,
\]

\[
M^2 = \frac{X^2}{2} - \left( \sigma r + \frac{C_1}{C_o} \right), \quad \bar{KE} + \bar{APE} = \frac{C_o}{2} \left( X^2 - \frac{\sigma}{r} (Y^2 + Z^2) \right) = C_2.
\]

\[
\bar{KE} = \frac{1}{2} \int \int \int (u^2 + w^2) dz dx,
\]

\[
\bar{PE} = - \int \int \int g\alpha (z\theta) dz dx,
\]

\[
\bar{APE} = - \frac{g\alpha H}{2\Delta T} \int \int \int (\theta)^2 dz dx.
\]
Closed-form Solutions with the Trigonometric and Elliptic functions

While waiting at the SFO airport:

- It is nonlinear, forced and non-dissipative.
- It is conservative for KE+PE and KE+APE, here KE, PE and APE are referred to as kinetic energy, potential energy, and available potential energy.
- Numerical solutions of energy conservative quantities (KE+PE and KE +APE) show dependence on time intervals and/or forcing parameters.

\[ \frac{d^2 X}{d \tau^2} + M^2 X = 0, \quad M^2 = \frac{X^2}{2} - \left( \sigma r + \frac{C_1}{C_o} \right). \]
Butterflies in the Lorenz Models

3D Lorenz Model with $r=25$

5D Lorenz Model with $r=25$

(b) 3D-NLM ($r=0$)

Nonlinear Oscillatory Solutions

$(0,0)$ is a stable critical point.
Diverged Trajectories

Nonlinearity may cause a frequency shift in the solutions of the control and perturbation runs.

The continuous frequency shifts lead to diverged trajectories.

Yic = 1
Yic = 1 + 0.001
Diverged Trajectories caused by Phase Shift

\[ \frac{d^2 X}{d\tau^2} + M^2 X = 0, \]
\[ M^2 = \frac{X^2}{2} - \left( \sigma r + \frac{C_1}{C_o} \right). \]
\[ Y^2 + \left( \frac{X^2}{2\sigma} \right)^2 = Y_{ic}^2, \]
\[ Y = Y_{ic} \cos(\phi), \]
\[ \phi = \int \int Y d\tau_1 \tau_2. \]
\[ X^2 = 2Y_{ic} \sigma \sin(\phi) \]
\[ \phi = \int^\tau X d\tau_0. \]
Ensemble Lyapunov Exponents (eLEs)

- $\Delta \tau = 0.0001$
- $N = 10,000,000$ (time steps), giving the total $\tau = 1,000$.
- $En = 10,000$ ensemble runs with Gaussian white noise as ICs
Reviewer’s Comments

Evaluation/Assessment of the Technical Progress of the Project

This is an extremely ambitious project of high value to NASA and the nation. The team is making progress on all tasks and I don’t see any stumbling blocks to achieving all the year 2 milestones. All aspects of the year two tasks were discussed.

Now about your question. I believe that it can as whether the system is conservative or not, trajectories must still diverge for chaos to be present. The fact that this is true for finite times I found out from my two PhD students who are working on orbital stability of exoplanets. They confirmed your statement about Hamiltonian chaos.
Butterflies in the Lorenz Models

3D Lorenz Model with $r=25$

5D Lorenz Model with $r=25$

(b) 3D-NLM ($r=0$)

$Y^2 = 1 - X^4/4\sigma^2$

3D-NLM ($r=25$)

Saddle point

When Sandy Meets Lorenz

National Central Univ. Jan. 16, 2015
Glasswinged Butterfly in the 3D-NLM

3D-NLM (r=25)

The wing pattern of a glasswinged butterfly
Energy Cycle in the 3D-NLM

Half of a big cycle

\[ (X_c, Y_c) = (\pm \sqrt{2\sigma r + X_0^2}, 0) \]

Linear
\[ \| APE_1 \| \geq \| APE_2 \| \]
Nonlinear
\[ \| APE_1 \| \leq \| APE_2 \| \]

\[ \sqrt{r/\sigma} X \]
\[ -X^2/2\sigma \]

a growth mode

a decay mode

Conservative Nonlinear oscillatory solutions
Nonlinear Oscillatory Solutions

A saddle point at (0,0) and nonlinearity

\[(X_c, Y_c) = (\pm \sqrt{2or + X_0^2}, 0)\]
A Summary of Shen (2014b)

5. Based on the energy analysis, an energy cycle with four different regimes is identified with the following four points: A(X,Y)=(0,0), B=(X_t, Y_t), C=(X_m,Y_m), D=(X_t, -Y_t). (X_t, Y_t)=(\sqrt{2\sigma r}, r), (X_m,Y_m)=(2\sqrt{\sigma r}, 0).

6. The conservative 3D-NLM can be analytically reduced into a 2D system, which is not chaotic (at infinite time).

7. A small but positive ensemble Lyapunov exponent over a total time period of (\tau=1,000) indicates the presence of diverged trajectories over a long but finite period of time in the 3D-NLM with r=0 and r\neq0. \rightarrow This suggests the appearance of Hamiltonian chaos.

8. The following factors may cause diverged trajectories: (1) nonlinearity alone (i.e., nonlinear feedback loop), which may continuously change the phase of the solution and (2) the inclusion of a heating term that could introduce a saddle point and interact with the nonlinear terms.
Lessons Learned from the Studies of Hurricane Sandy and Lorenz models

- Accurate representations of environmental flows near a saddle point are crucial for capturing the northwestward movement of Hurricane Sandy prior to its landfall.
- Nonlinearity and the appearance of a saddle point can lead to chaotic responses in a non-dissipative Lorenz model, which is a conservative and has nonlinear oscillatory solutions.
- High (spatial) resolution can improve the representation of environmental flows near a saddle point where the gradient of wind directions is large.
- A lat-lon grid system, which non-uniform, may have advantage of simulating Hurricane Sandy as its grid spacing decreases when Sandy moved northward?
- Factors in determining the phase speed of systems (e.g., an upper trough and/or vortex itself) may determine the eastward or westward movement of Sandy near the saddle point; these factors include grid spacing, initial conditions and physics parameterizations etc, (some of which may have cumulative impacts).
• 3D-NLM, fixed saddle point in the phase space
• Sandy, moving fixed saddle point in the spatial space, (part of the saddle point)
Summary

Current Project NASA AIST CAMVis: 2012-2015

MAP: Multiscale Analysis Package
HHT: Hilbert Huang Transform/
PEEMD
SAT: Stability Analysis Tool

- to what extent can large-scale flows
determine the timing and location of TC
genesis? MAP/HHT
- to what extent can resolved small-scale
processes impact solutions stability (or
predictability)? MAP/SAT
Short-term Climate Simulations of AEWs and AEJ

